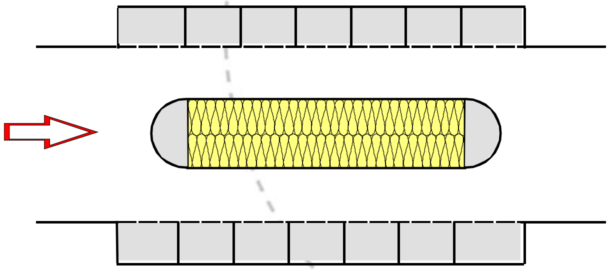
Norsk Akustisk Selskap



Sirkulære lydfeller. Kanalvegger med mikroslisser

[Circular duct silencers using duct walls with micro-slits]

T.E. Vigran and T. Haugen

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Outline

- General theory of sound propagation in lined circular and annular ducts
- Special case using linings of plates with micro-slits.
- Measurement method
- Results – measurement and prediction

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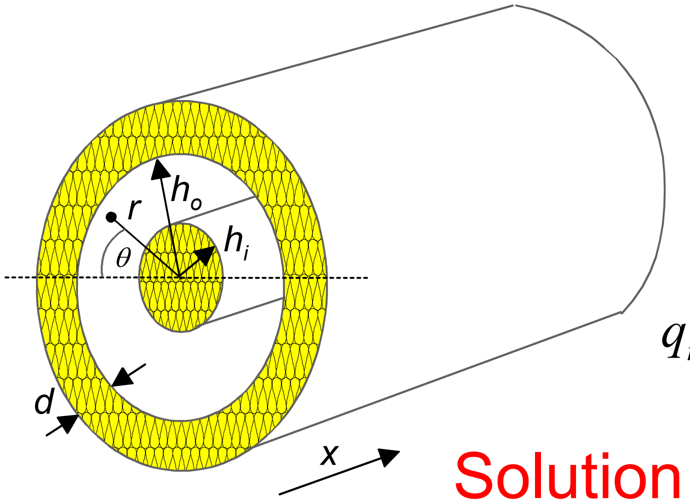
Dissipative type of silencers, following F.P.Mechel



Modal sound pressure

$$p_m(r,\theta,x) = A q_m(r,\theta)\,\mathrm{e}^{-\Gamma x}$$

$$q_m(r,\theta) = \cos(m\theta)\left[\mathrm{J}_m(\varepsilon_m r) + b\,\mathrm{Y}_m(\varepsilon_m r)\right] \qquad m = 0,1,2,....$$

Solution of the wave equation

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \Gamma^2 + k_0^2\right] q(r,\theta) = 0$$

Propagation coefficient gives the attenuation

$$\text{Attenuation(dB/m)} \approx 8.69 \cdot \mathrm{Re}\left(\Gamma\right)$$

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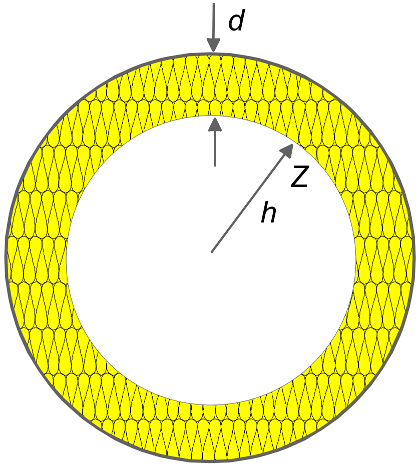
Lined duct, i.e. without pod



Particle velocity (radial component)

$$v_r(r,\theta,x) = \frac{\mathrm{j}\,A\varepsilon_m}{k_0 Z_0}\cos(m\theta)\,\mathrm{J}'_m(\varepsilon_m r)\,\mathrm{e}^{-\Gamma x}$$

The impedance at *r* = *h* must match the impedance *Z* of the duct wall. For *m* = 0:

$$\varepsilon_0 h_o \cdot \frac{\mathrm{J}_1(\varepsilon_0 h_o)}{\mathrm{J}_0(\varepsilon_0 h_o)} = -\mathrm{j}\cdot\frac{k_0 h_o}{Z_n}$$

giving

$$\Gamma = \sqrt{\frac{(\varepsilon_0 \cdot h_0)^2}{h_o^2} - k_0^2}$$

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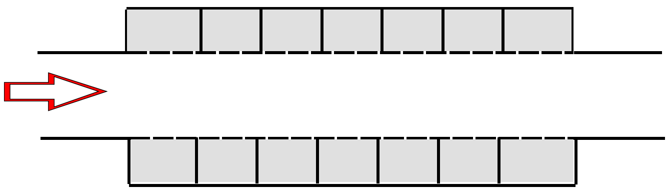
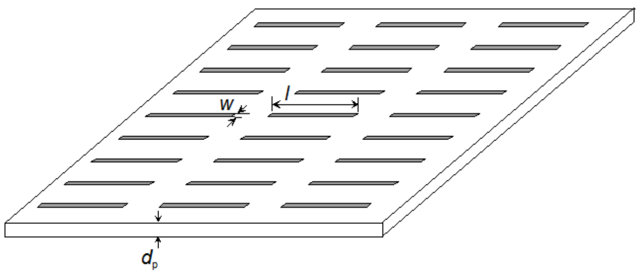
Now for the duct walls with micro-slits



Air-filled cavity

$$Z_{an} = 1 \text{ and } \Gamma_a = \mathrm{j} k_0$$



Micro-slit plate

$$Z_n = \frac{1}{Z_0 \sigma}\left[Z' + \mathrm{j}\rho_0 \omega (2\Delta d_\mathrm{p}) \right]$$

where (with a simplified model):

$$Z' = \mathrm{j}\rho_0 \omega d_\mathrm{p} \left(1 - \frac{\tan(\lambda_s / \sqrt{\mathrm{j}})}{\lambda_s / \sqrt{\mathrm{j}}} \right)^{-1} \quad \text{where} \quad \lambda_s = \frac{w}{2}\sqrt{\frac{\omega \rho_0}{\mu}}$$

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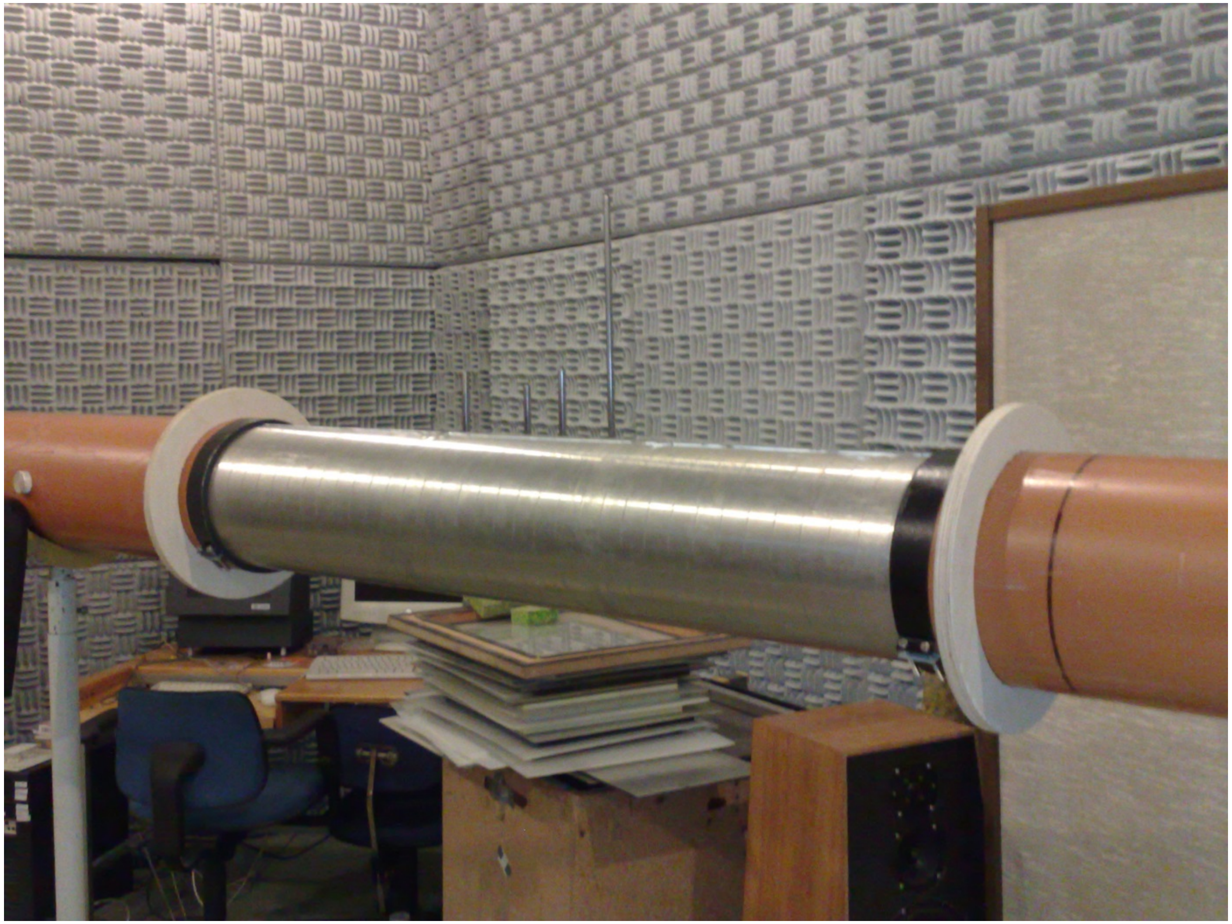
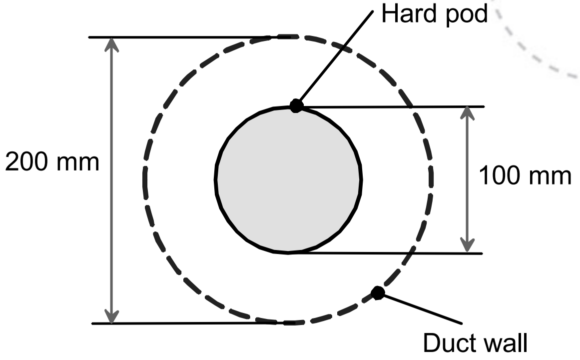
An interesting case?





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An interesting case?

In this case we need to combine the impedance of the duct wall

$$Z_n = \frac{1}{Z_0 \varepsilon}\left[Z' + \mathrm{j}\,\rho_0 \omega (2\Delta d_{\mathrm{p}}) \right]$$

with the air around deduced from the radiation impedance of a vibrating cylinder. Assuming zero mode, we get

$$Z_{an} = \frac{\left(Z_{\mathrm{rad}}\right)_{0,0}}{Z_0} = \mathrm{j}\cdot \frac{\mathrm{J}_0(k_0 h) - \mathrm{j}\mathrm{Y}_0(k_0 h)}{\mathrm{J}_1(k_0 h) - \mathrm{j}\mathrm{Y}_1(k_0 h)}$$

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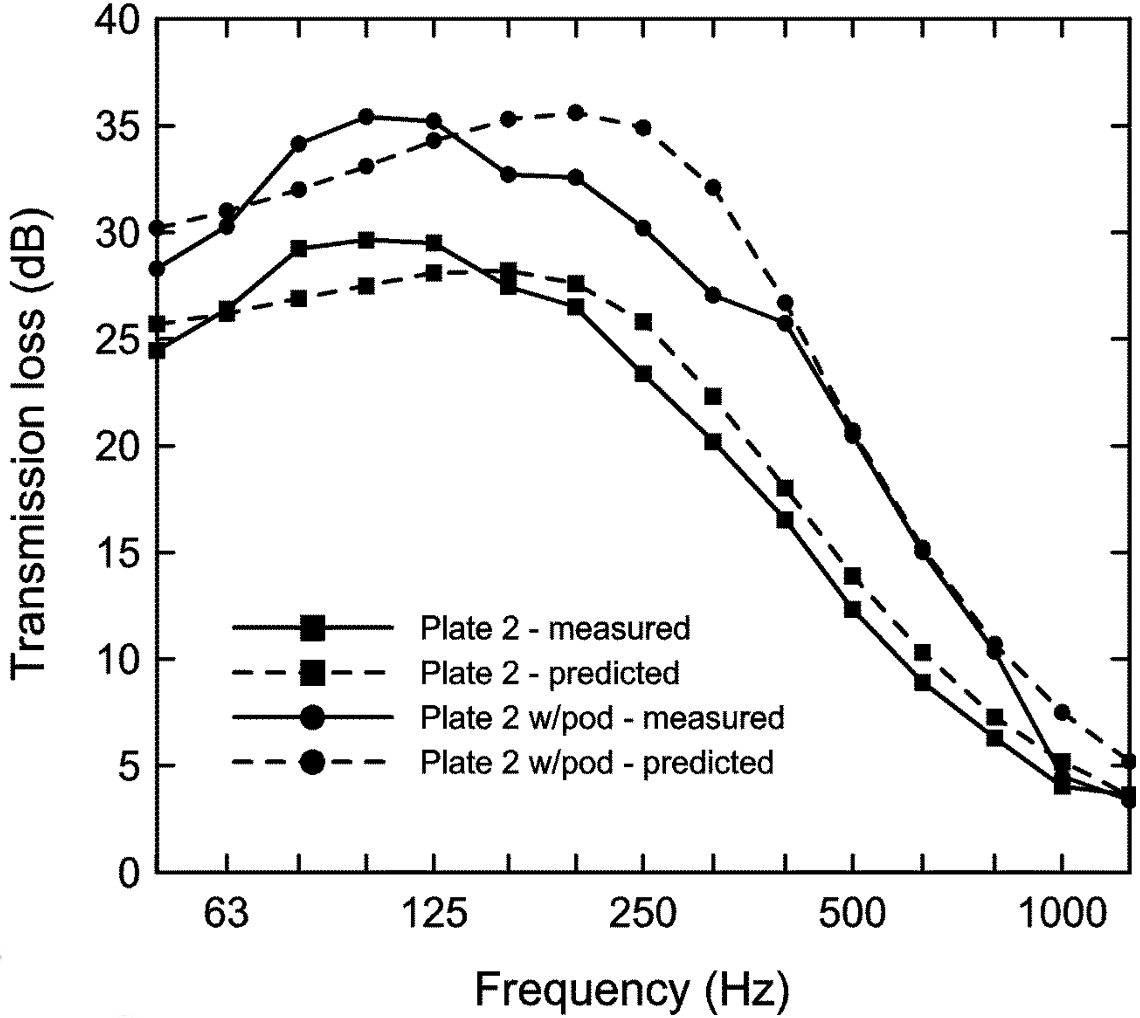
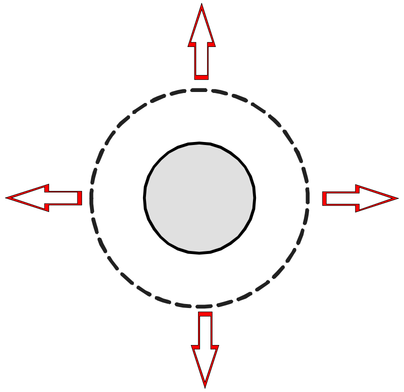
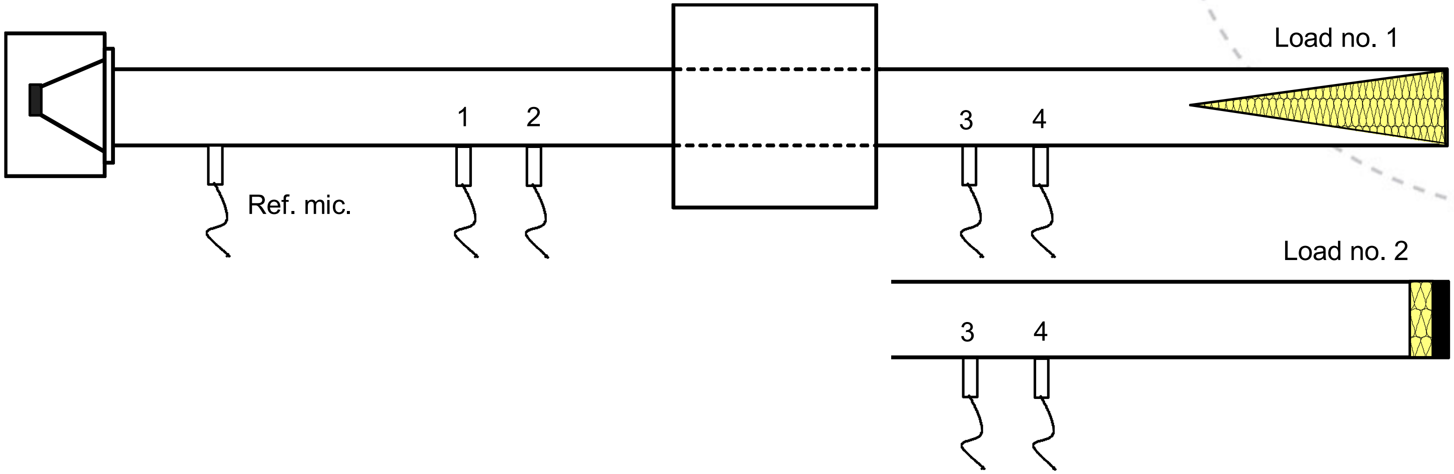
An interesting case?





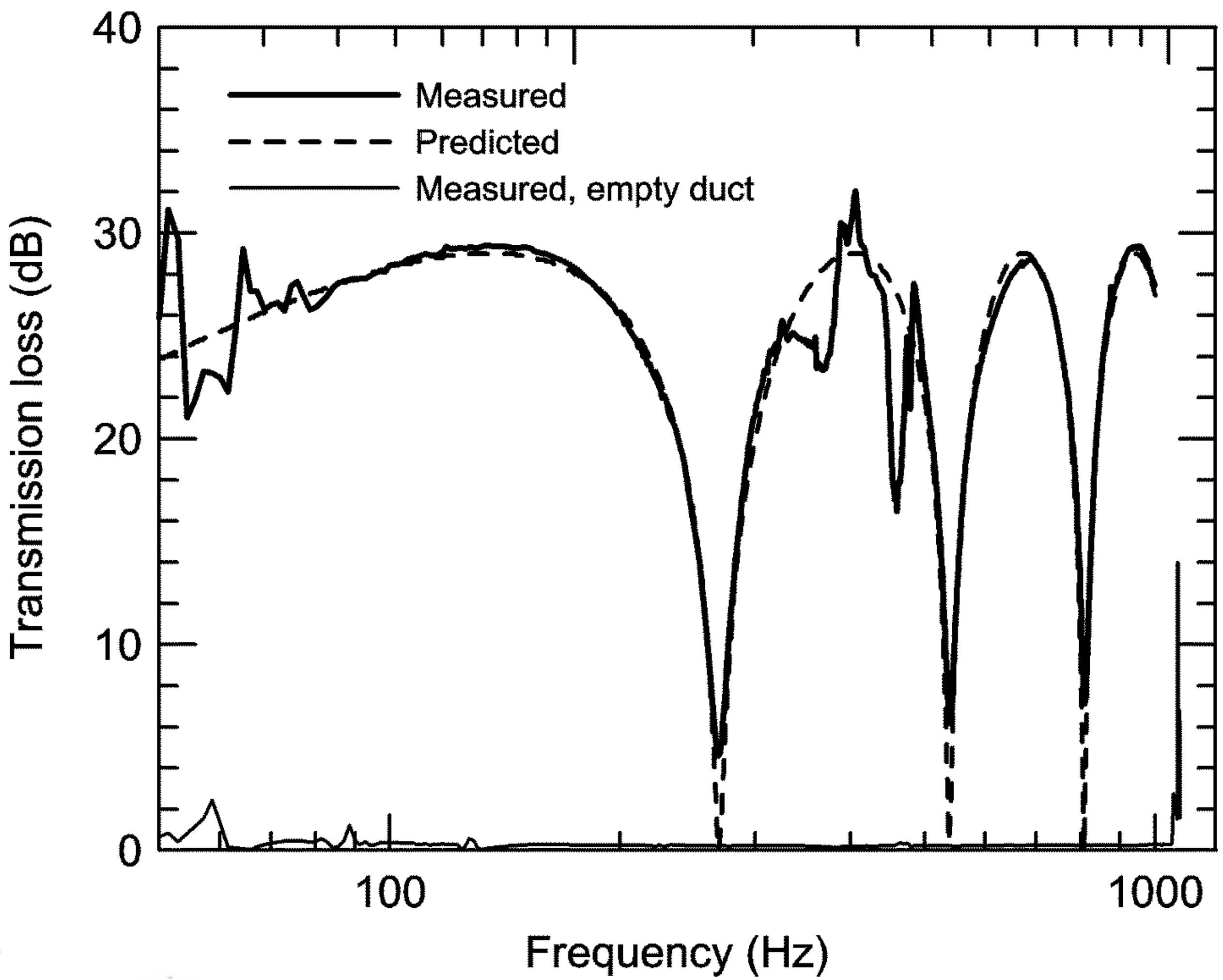
How is this measurements done?

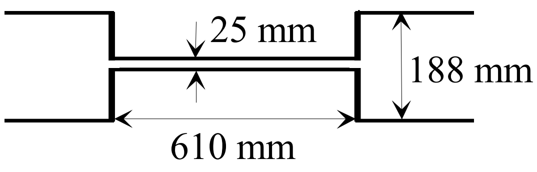


Two different two-load methods used:

1. Y. Salissou and R.Panneton, J. Acoust.Soc.Am. 125 (2009)
2. ASTM E 2611-09

Testing set-up





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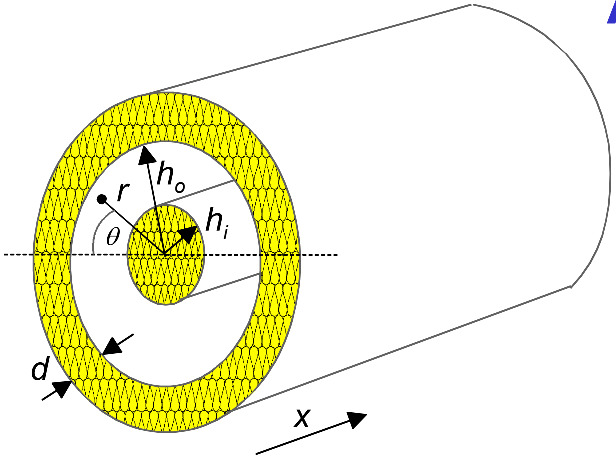
Annular duct (1)



Particle radial velocity with pod

$$v_r(r,\theta,x) = \frac{\mathrm{j}\,A\varepsilon_m}{k_0 Z_0}\cos(m\theta)\left[\mathrm{J}'_m(\varepsilon_m r) + b\,\mathrm{Y}'_m(\varepsilon_m r)\right]\mathrm{e}^{-\Gamma x}$$

Knowing the impedances of duct and pod surfaces, we must solve for amplitudes A and B (=A*b)

$$\frac{\mathrm{j}\varepsilon_m}{k_0}\frac{A\,\mathrm{J}'_m(\varepsilon_m h_{o,i}) + B\,\mathrm{Y}'_m(\varepsilon_m h_{o,i})}{A\,\mathrm{J}_m(\varepsilon_m h_{o,i}) + B\,\mathrm{Y}_m(\varepsilon_m h_{o,i})} = \pm\frac{1}{(Z_n)_{o,i}}$$

Here again for the special case of *m* = 0

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Annular duct (2)

This ends up solving the equation

$$\alpha z^2\left[J_1(z)\,Y_1(\alpha z)-J_1(\alpha z)\,Y_1(z)\right]$$
$$-\,jU_o\alpha z\left[J_0(z)\,Y_1(\alpha z)-J_1(\alpha z)\,Y_0(z)\right]$$
$$+\,jU_i z\left[J_1(z)\,Y_0(\alpha z)-J_0(\alpha z)\,Y_1(z)\right]$$
$$+U_oU_i\left[J_0(z)\,Y_0(\alpha z)-J_0(\alpha z)\,Y_0(z)\right]=0 \qquad z=\varepsilon\cdot h_0 \text{ and } \alpha=\frac{h_i}{h_0}$$

and where the *U's contain the impedances*

$$U_o=\frac{k_0h_o}{Z_{on}} \qquad \text{and} \qquad U_i=\frac{k_0h_i}{Z_{in}}$$

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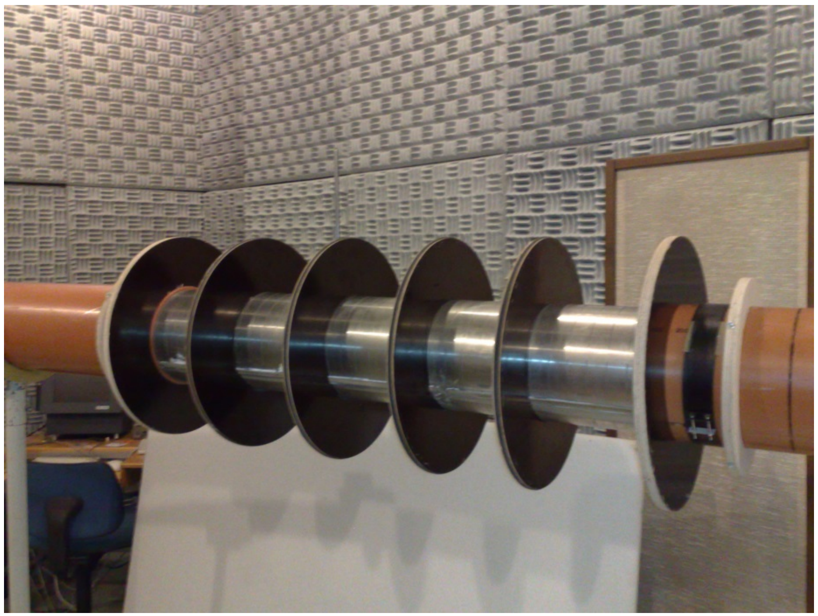
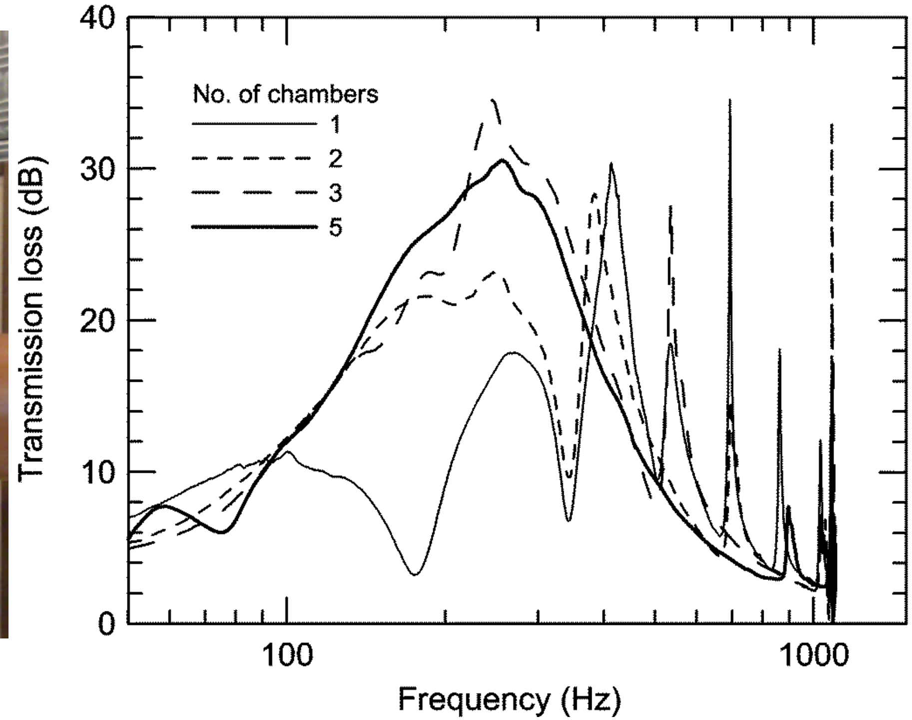
Measurements – The influence of chambers





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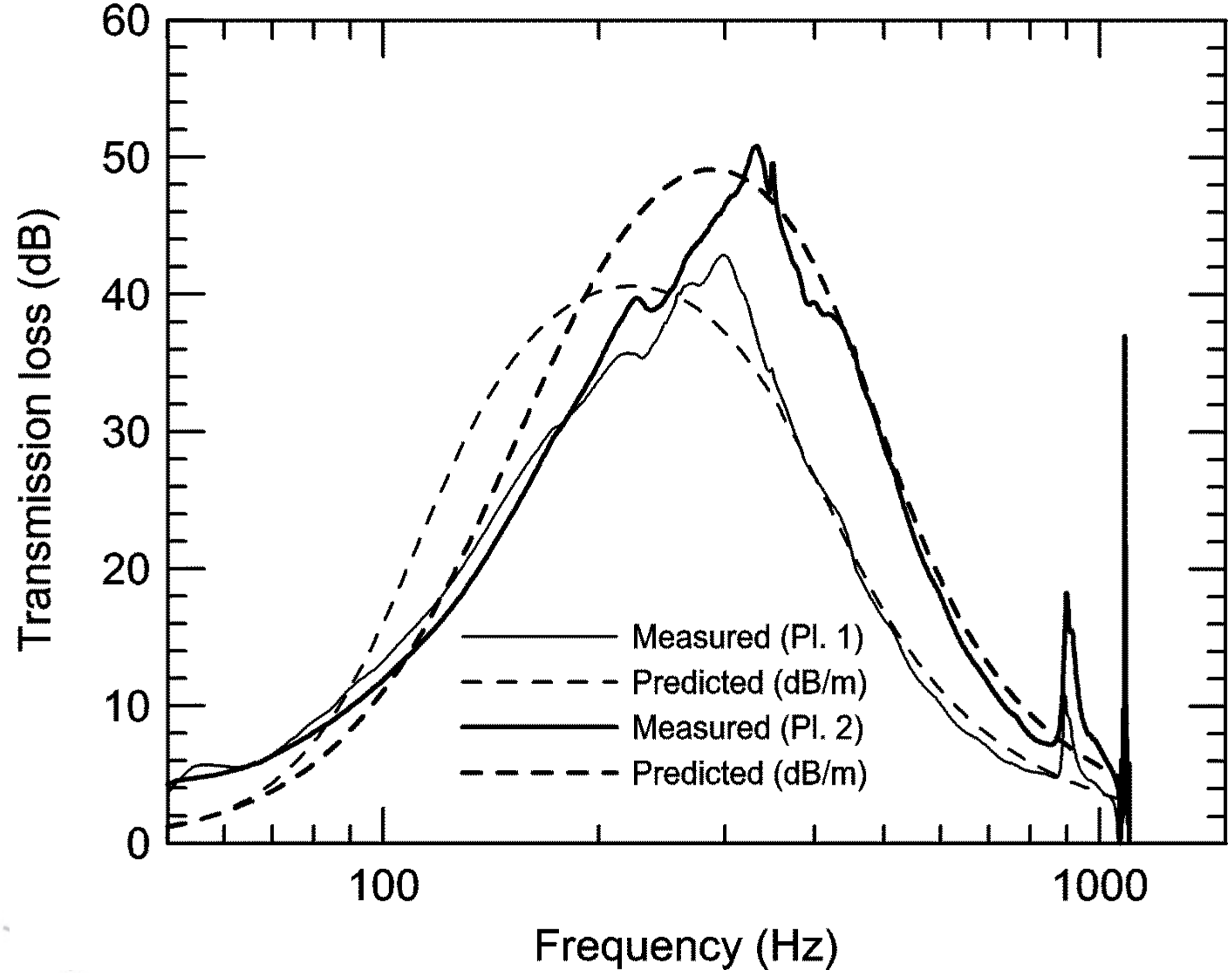
Measurements – Including a hard pod of diameter 10 cm. Two different plates.



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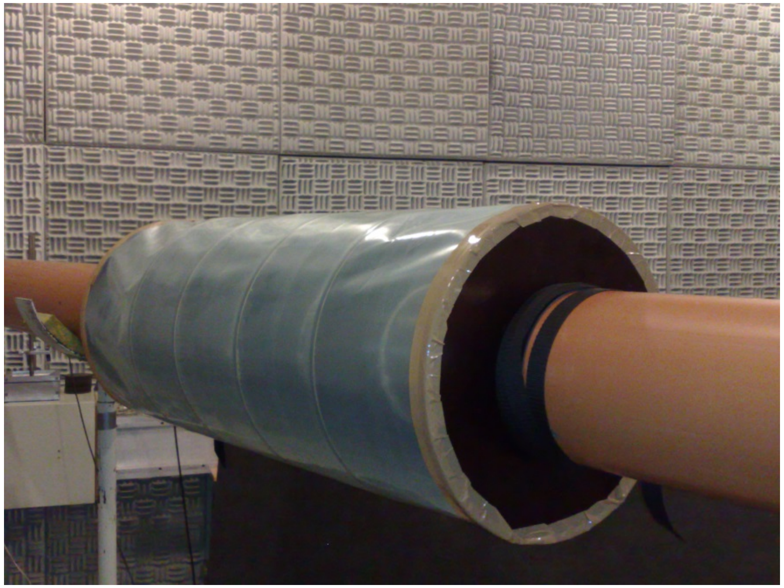
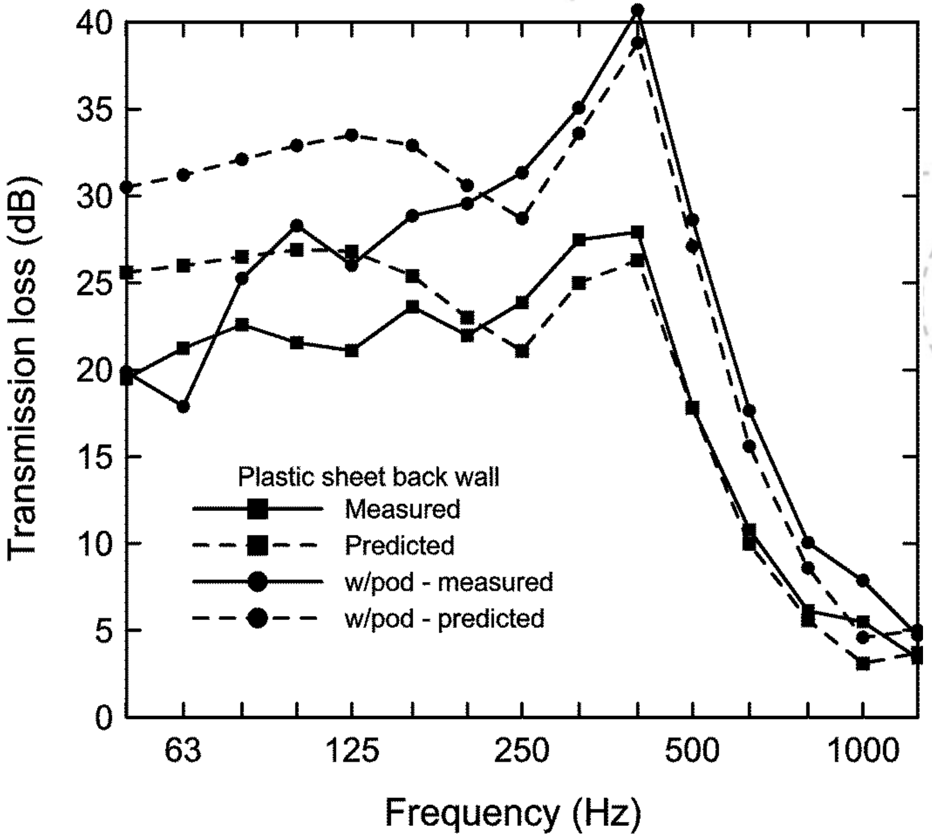
A special case: silencer with a soft outer duct wall





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Some conclusions

- Silencers with linings of micro-slitted plates may be adjusted to cover a relatively broad frequency band
- Low frequency attenuation is substantially increased using a soft outer wall or no outer wall
- The pressure loss of such silencers is negligible compared with conventional silencers, however not shown here.

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Thank you for your attention!