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Comparison of Different Computational Methods for Acoustic Wave Propagation

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In this paper, we discuss the application of available methods for calculation of acoustic wave propagation that are available in COMSOL Multiphysics®. In particular, we present wave-based implicit methods for frequency-domain simulations with modern iterative solvers; a dG-FEM (discontinuous Galerkin) time explicit, higher-order method, for transient simulations; and also, a ray tracing method with a hybrid FEM approach, where the source and its vicinity are modeled in detail. These methods are highlighted and compared with regards to accuracy and performance. The feasibility for different applications, with pros and cons of these methods, are discussed with examples taken from room acoustics applications, including a car cabin.

1 Introduction

When modeling room acoustics, that is, concert halls, but also other closed spaces like car cabins or recording studios, the approach has typically been to either focus on the high frequency limit or the low frequency modal behavior. The methods concerned with the high frequency limit are based on various geometrical acoustics approximations, like ray tracing and image source techniques, but also on energy methods that solve a diffusion equation for the sound energy [1,2,3]. Analytical approaches are based on a truly diffuse field assumption and include the well-known Eyring and Sabine equations. In the low frequency range, where the modal behavior dominates, analytical expressions exist for the sound field in simple room shapes [1,2,3]. For more complex shapes and boundary conditions, it is common to solve the Helmholtz equation using numerical methods like, for example, the finite element method (FEM) or the boundary element method (BEM).

The numerical methods used in either the high or low frequency range have various challenges that are related to the underlying physical assumptions made.

The low frequency behavior typically involves solving the full wave problem in the frequency domain. This can lead to models that are very demanding from a computational point of view: as the frequency increases the model size increases. This is because the computational mesh (the spatial discretization) must resolve the wavelength in the problem. At low frequencies, the phase information is critical for accurately solving the problem. Thus, wall properties cannot only be described through an absorption coefficient, but the phase change of reflected waves must be included. Walls and boundaries must be described through a complex-valued impedance, or even by modeling the absorber configuration in full detail.

Ray tracing type methods are high frequency approaches. They assume that the wavelength is much smaller than any characteristic geometry scale. The sound field is assumed to locally behave like a plane wave. For practical applications, this condition is often violated, and models are solved at too low frequencies, for example, when modeling car cabin acoustics. Moreover, the frequency resolution in the model is defined through frequency bands; the source and wall (random incidence) absorption coefficient are often defined only in these bands. Sources are also often defined through their far-field radiation pattern (beyond the Rayleigh radius) only. Diffraction effects can be included but are often ignored.

Common to both approaches above is that the proper definition of boundary conditions is challenging [4,5,6]. Many surfaces are only defined through either the normal impedance (for FEM based models) or their diffuse field absorption and scattering coefficient. The angle dependency of the absorption/impedance, as well as the frequency dependency, is not always known. Surfaces may be assumed to be locally reacting while they in fact have an extended reaction behavior. In this paper, we suggest how surfaces can be treated in a more consistent manner. In the full wave models, boundaries need to be modeled in detail, for example, the air volume and porous panel in a lowered ceiling (resonant absorber) needs to be included fully. In ray tracing, a sub-model of the lowered ceiling can be set up to extract the angle dependent absorption; this value is in turn used in the wall conditions where the absorption is made dependent on both frequency and angle of incidence. Measured values of surface properties are still a topic of research.

When the ray tracing method is used to simulate a complex listening environment like a car cabin, the proper source definition can also be challenging. The radiation characteristics of a loudspeaker will change drastically when it is placed near a scattering surface, like the wind screen or dashboard. In this case, the loudspeaker source should not be characterized by its free field radiation pattern, but by its radiation pattern in the environment where it is located. A method to couple the local radiation behavior modeled with FEM to a ray tracing model is also presented here.

As mentioned, ray tracing is often used at low frequencies where it is actually not applicable. This stems from practical purposes: solving the full wave model in the frequency domain up to a frequency where ray tracing is applicable can be expensive. However, with new iterative solver strategies, as well as improved hardware, it is now possible to solve models much higher in frequency. The Helmholtz equation describing the frequency domain is in this article solved using the Acoustics Module of COMSOL Multiphysics® [8,9]. An iterative method based on a geometrical multigrid preconditioner with a complex shifted Laplacian (CSL) method is used. This allows solving room acoustic models much higher in frequency and thereby switching to ray tracing at a higher frequency where the method is applicable.

It is often interesting to stay in the frequency domain for the low frequencies. This makes it simple to include frequency dependent boundary conditions (impedance conditions that include the phase shift at walls) as well as porous materials. The frequency domain transfer function can be transformed into an impulse response using an inverse Fourier transform.

The final approach discussed here involves solving the room acoustic problem with a full wave approach in the time domain. This approach is made possible by the recent development and implementation in commercial software of numerical methods like discontinuous Galerkin (dG) higher order finite elements, spectral element methods, or the more classical finite difference time domain method [5]. In this paper the use and application of the dG time explicit method is also discussed. The Acoustics Module of COMSOL Multiphysics® [8,9] offers several physics interfaces that are based on dG-FEM. The method is higher order in both time and space (removes the problem of numerical dispersion) and it is based on an explicit matrix free method. This makes the method well suited for distributed computing. Solving in the time domain allows direct computation of the room response for any input signal (band passed or broadband signal). However, solving in the time domain does come with certain challenges, in particular proper definition of frequency dependent boundary conditions. Research is done in this regard [7, 11-15].

2 Modeling Methods

The three methods studied in this paper are now shortly introduced.

2.1 Finite Element Method

The Acoustics Module of COMSOL [9] has an interface for solving pressure acoustics in the frequency domain. The finite element method is used to solve the Helmholtz equation:

$$\nabla \cdot \left(\frac{1}{\rho_c}\nabla p\right) + \frac{k^2}{\rho_c} p = 0 \quad k = \frac{\omega}{c_c} \tag{1}$$

where p is the acoustic pressure, $\omega = 2\pi f$ is the angular frequency, k is the wave number, ρ_c is the density, and c_c is the speed of sound. Both density and speed of sound can be complex valued to define porous domains. Several built-in porous model options are defined in the Acoustics Module. At boundaries, a normal impedance boundary condition is typically used.

2.2 Hybrid Ray Tracing-FEM Approach

The Acoustics Module of COMSOL [9] has an interface for modeling acoustic ray tracing. The interface is very versatile. For modeling sources, it has the classical options to release rays from a point with a given spatial radiation pattern. This option is used for modeling speakers in large rooms. Rays can also be released from a boundary with a given uniform distribution of intensity and release directions. This functionality is used to set up more realistic sources. The ray source distribution can be based on solving a pressure acoustics model in a limited space using the FEM method. This could be a loudspeaker including scattering surfaces. An example is depicted in Figure 1 for a speaker located near the A-pillar in a car. The spatial intensity distribution (integrated to distribute the acoustics power) and intensity vector from the local model define the ray acoustics source. This results in a hybrid ray tracing-FEM approach, but not in the usual sense where the two methods are used in different frequency ranges. After release, the rays are free to propagate, also in the FEM sub-model domain.

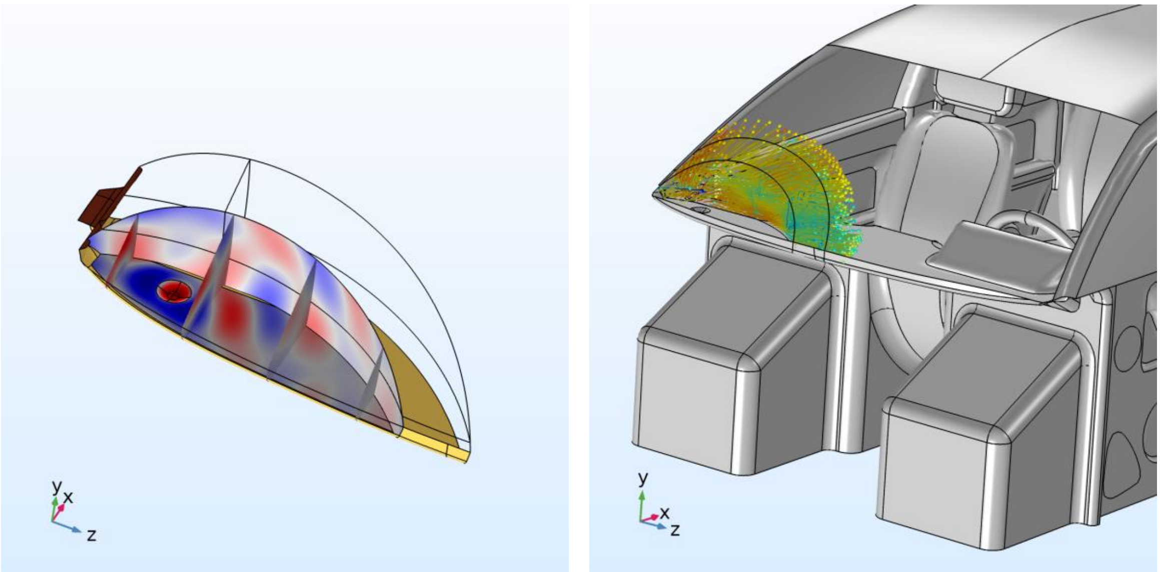

Figure 1: *(left) Acoustic field generated by the loudspeaker including windscreen and dashboard, and (right) the released rays that reconstruct the source radiation characteristics.*

Wall properties are modeled in the FEM domain as normal impedance boundary conditions. The ray acoustics model can either consider the wall as purely absorbing surfaces (typically defined in bands) or as a complex valued impedance can be defined. In the latter case the phase is solved for along the rays and properties are defined for a single frequency. The absorption or impedance can be made dependent on both frequency (band or single frequency) and the angle of incidence. Volumetric absorption, which is important at high frequencies, can also be included.

2.3 Discontinuous Galerkin, Time Explicit

The Acoustics Module of COMSOL [9] has a dG-FEM interface for solving pressure acoustics in the time domain. The dG time explicit method solves the first order formulation of the wave equation:

$$\frac{1}{\rho c^2}\frac{\partial p}{\partial t} + \nabla \cdot \mathbf{u} = 0$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \nabla p = 0 \qquad (2)$$

where p is the acoustic pressure, $\mathbf{u}$ is the acoustic velocity, ρ is the fluid density, and c is the speed of sound. In this formulation the material properties are real valued.

3 Application Examples

In the following sections, four examples are presented that use the discussed methods and applications. In the first example, a 144 m^3 room with a lowered ceiling is modeled using FEM and ray acoustics. Then three examples show various examples of the FEM method, the hybrid ray-FEM approach, and the dG-FEM method applied to a car cabin acoustics problem. All models are run on a 2 Socket Intel® Xeon® CPU E5-2687W v3 @ 3.10GHz (20 cores in total) machine with 128 GB of RAM.

3.1 Room Acoustic with an Extended Reaction Boundary

As an example of a non-trivial room acoustics problem, let us look at an example of a classroom with a volume of 144 m^2 with a lowered ceiling. The ceiling consists of a resonator volume with a depth of 20 cm with a porous plate in front. The porous plate has a thickness of 2 cm and is made of some compacted fibrous material with a flow resistivity of $50{\cdot}10^3$ Pa·s/m^2. The surface impedance of the ceiling does not follow a classical locally reacting surface impedance; it has a so-called extended reaction behavior. The other boundaries of the room have a low absorption coefficient applied.

This results in a situation where the acoustic field will not be ideally diffusive. The acoustics of the room when excited by a perfect omnidirectional source are modeled with two methods. For the low (modal) to medium frequency behavior up to 500 Hz, the model and the ceiling is solved in detail with a full wave-based FEM simulation in the frequency domain, using the Pressure Acoustics interface of COMSOL [9]. Above the estimated Schroeder frequency of the room (of about 160 Hz), the acoustics are also simulated using the Ray Acoustics interface of COMSOL [9]. For the ray tracing simulation, it is necessary to know the angle and frequency dependent absorption properties of the ceiling. This value is computed using a sub-model of the ceiling configuration running a sweep over frequencies and angle of incidence of plane waves, see Figure 2.

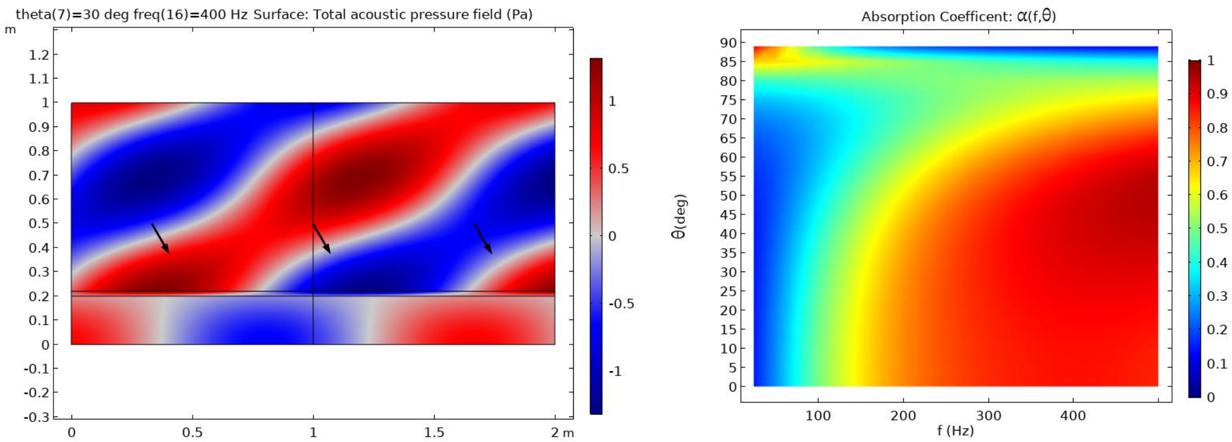

Figure 2: *(left) Pressure distribution in the sub-model of the ceiling configuration. The arrow shows the direction of the incident pressure field. (right) Absorption coefficient as function of frequency and angle of incidence.*

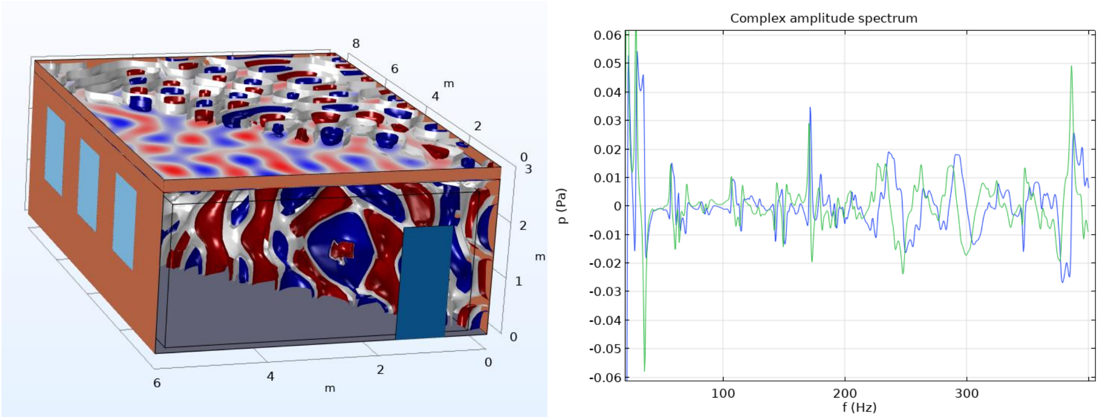

Figure 3: *(left) Pressure iso-surfaces in the room at 400 Hz (seen above and below the porous layer) with walls, floor, windows, and entrance door highlighted. (right) The real and imaginary part of the pressure at the receiver.*

The source is located at (x,y,z) = (2 m, 2 m, 1 m) and the receiver at (x,y,z) = (7 m, 5 m, 1 m). The resulting pressure distribution at 400 Hz is depicted in Figure 3 (left) while the frequency response (real and imaginary part of the pressure) is depicted in Figure 3 (right). The frequency response is run through an inverse Fourier transform and filtered into octave bands. Some selected results are shown in Figure E.

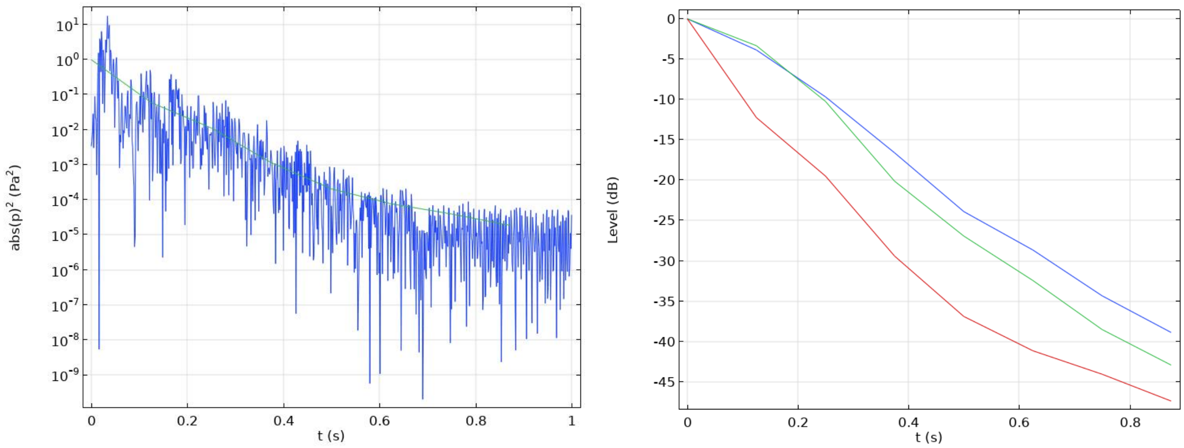

Figure 4: *(left) Energy decay curve of the 250 Hz Octave band with Schroeder integration shown. (right) Level decay curves for the 62.5 Hz (red), 125 Hz (green), and 250 Hz (blue) octave bands.*

The results of the ray tracing simulation are shown after 7 ms in Figure 5 (left). In the ray tracing model, the ceiling and air cavity is not modeled. In this case, an angle and frequency dependent absorption coefficient $\alpha(f,\theta)$ is defined. The level decay curves based on the ray tracing model is depicted in Figure 5 (right). The level drops 20 dB in 0.4 s, which is the same as seen in the level decay curve computed with the full wave model, the blue curve in Figure 4 (right).

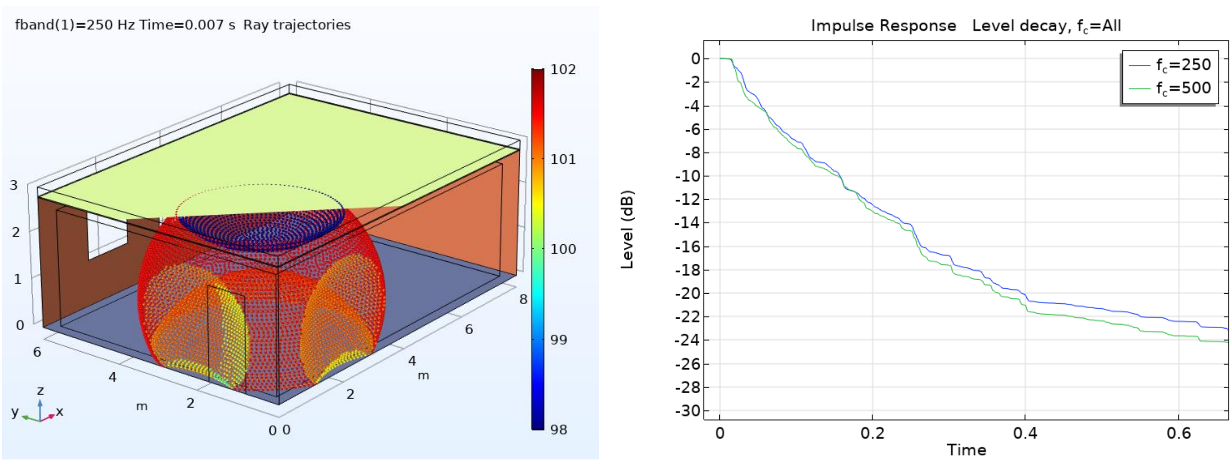

Figure 5: *(left) Ray positions after 7 ms. The colors are the local SPL levels. (right) Level decay curves for the 250 Hz and 500 Hz octave bands.*

3.2 Low Frequency Car Cabin Acoustics

The acoustic field and transfer functions in a generic car cabin geometry is modeled solving the Helmholtz equation using FEM. The Pressure Acoustics, Frequency Domain interface of the Acoustics Module in COMSOL [9] is used. The geometry depicted in Figure 6 is used here and in the next two examples. The car cabin is equipped with two tweeters (left and right) and two woofer/mid frequency speakers (left and right). For simplicity, the speaker is modeled as a constant velocity source, but for a more realistic source the boundary can be coupled to a lumped model (defined through the Thiele-Small parameters). An absorption coefficient is defined for the windows, dashboard, and door lining. An impedance condition is applied for the roof trim and floor/carpet that represents a thin porous layer, and a measured complex valued impedance [10] is applied to represent the leather seats.

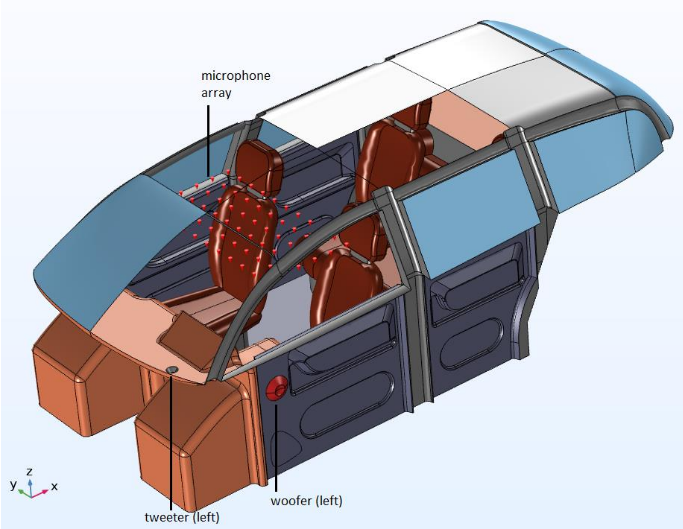

Figure 6: *Car cabin geometry used for the simulation examples.*

For the low frequency behavior, the left woofer is used as a source. The response of the car cabin is depicted in Figure 7. On the left, the sound pressure level distribution at 1 kHz is depicted, and on the right, the response in two microphone positions is shown. The model is solved from 10 Hz to 1 kHz with a 2 Hz resolution. The computation time is about 1 h and required 8 GB of RAM ($330{\cdot}10^3$ DOFs). In this case, a direct MUMPS solver is used as it is efficient for small to medium sized problems. The solver in COMSOL 5.6 [8] uses a new compression technology that gives a memory and computation time reduction of about 20-25 % compared to earlier versions.

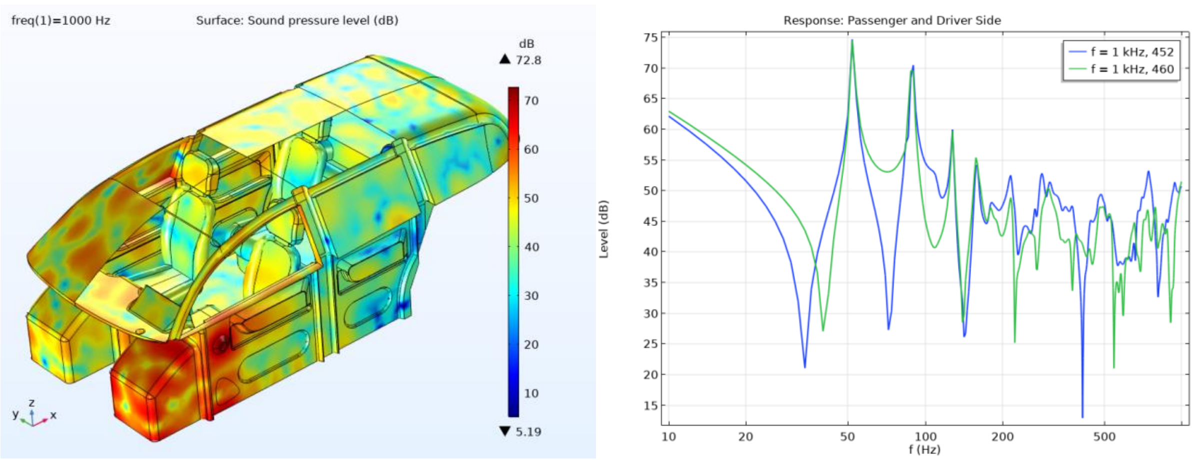

Figure 7: *(left) Sound pressure level distribution at 1 kHz, and (right) response measured on a microphone on the passenger and driver side.*

The car cabin model is also solved at a few selected higher frequencies to get an idea of the time consumption necessary to increase the frequency range for the frequency domain wave model. Solving at 2 kHz and 3 kHz took 2 min 10 s and 13 min, and required 15.8 ($2.0{\cdot}10^6$ DOFs) and 37.3 ($6.7{\cdot}10^6$ DOFs) GB of RAM, respectively. Both used the iterative solver with a geometric multigrid preconditioner with CSL contributions. For comparison, solving the model at 2 kHz with a direct solver took 3 min but required 45.5 GB of RAM. The model was also solved at 7 kHz on the 120 GB fat node of an older cluster. In this case, the model solved in about 10 h and required 105 GB of RAM, solving 83.5 million DOFs. The CSL method ensures convergence and increases convergence speed as the frequency is increased. The sound pressure level distribution at 3 kHz and the pressure distribution at 7 kHz are depicted in Figure 8. The 250 Hz octave band filtered impulse response (IR) is depicted in Figure 9. The IR is recovered using an inverse Fourier transform.

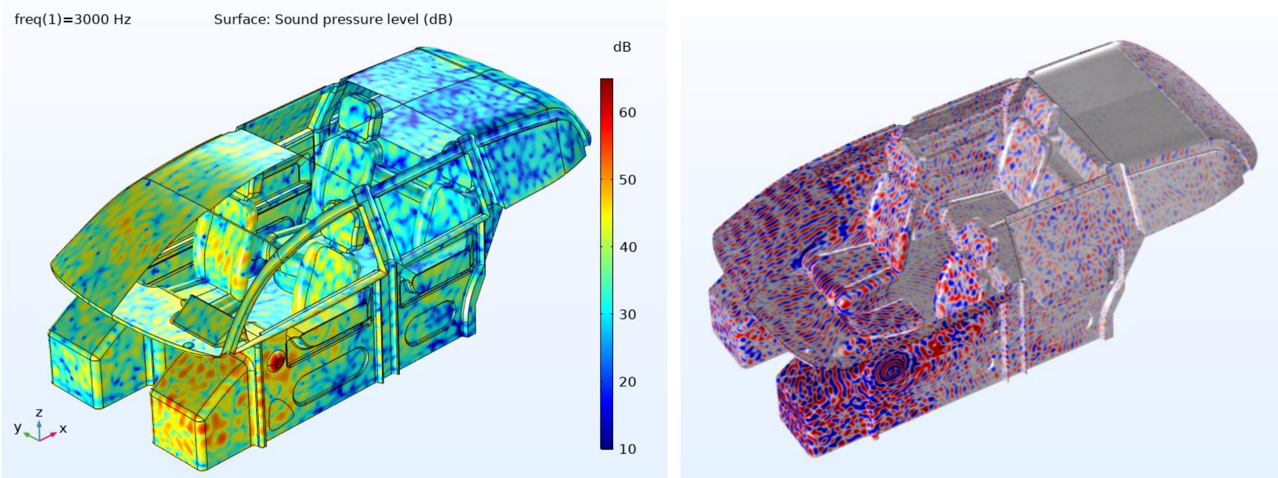

Figure 8: *(left) SPL distribution at 3 kHz, and (right) pressure distribution at 7 kHz (notice that the woofer is still being used as a source).*

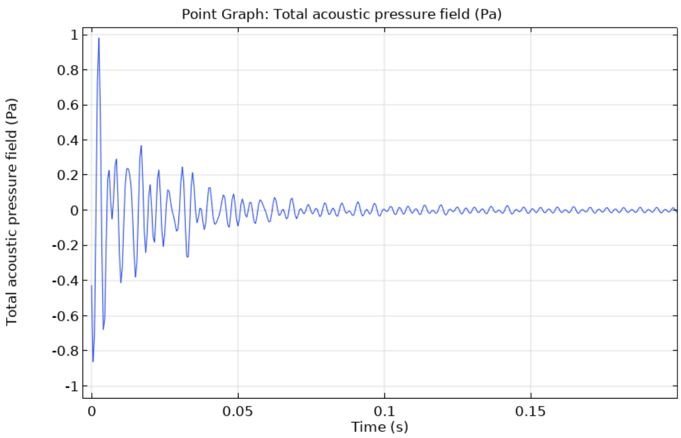

Figure 9: Impulse response of the 250 Hz octave band.

3.3 High Frequency Car Cabin Acoustics with Hybrid Ray-FEM Source

The method described in section 2.2 is used to model the sound field generated in a car cabin by a tweeter located close to the A-pillar. The source can be seen in Figure 1. For validating and assessing the hybrid FEM-ray source approach, the setup is solved with three approaches: the hybrid FEM-ray method; a classical ray tracing approach where the source is modeled as a point source (with a free-field radiation pattern); and finally, a full wave model as discussed above in section 3.2. The comparison of the three methods is carried out visually by computing the SPL distribution on the four sets in the car at 3000 Hz. The results can be seen in Figure 10. The classical ray tracing model results (bottom) show that the main part of the energy is focused to the driver seat. This is not consistent with the full wave method (top) or the hybrid approach results (middle).

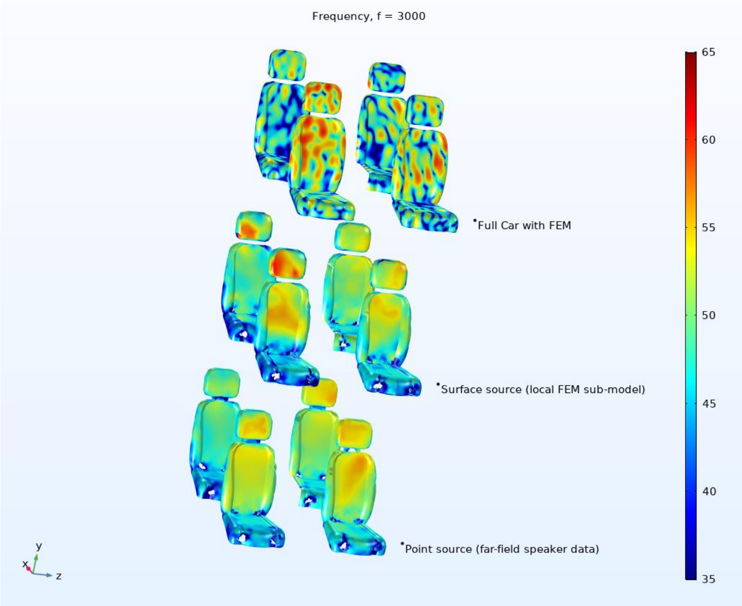

Figure 10: *Sound pressure level distribution at the seat surfaces for the three different modeling approaches*.

3.4 Car Cabin Acoustics with Full-Wave Transient dG-FEM

In this last example, the acoustic field inside the generic car cabin geometry is modeled solving the linearized Euler equations using the dG-FEM time explicit method (see section 2.3). The Pressure Acoustics, Time Explicit interface of the Acoustics Module in COMSOL [9] is used. The method is very memory lean and well suited for efficient distributed computing. A Gaussian modulated pulse with a center frequency at 3 kHz is emitted by the tweeter located in the dashboard (see Figure 6). The propagation is depicted at three time instances in Figure 11. Solving in the time domain has the advantage of directly producing an impulse response.

The model solves the $123 \cdot 10^6$ DOFs distributed on 16 nodes with 2 x Intel(R) Xeon(R) Platinum 8260 CPU at 2.40 GHz. The solution time was 3 days and 3 h and only required about 50 GB of RAM. For the current setup, the surfaces are described using a constant valued resistive impedance condition. More advanced frequency dependent impedance conditions can be defined solving a system of ODEs at the boundaries, see references [11-14], and also reference [15].

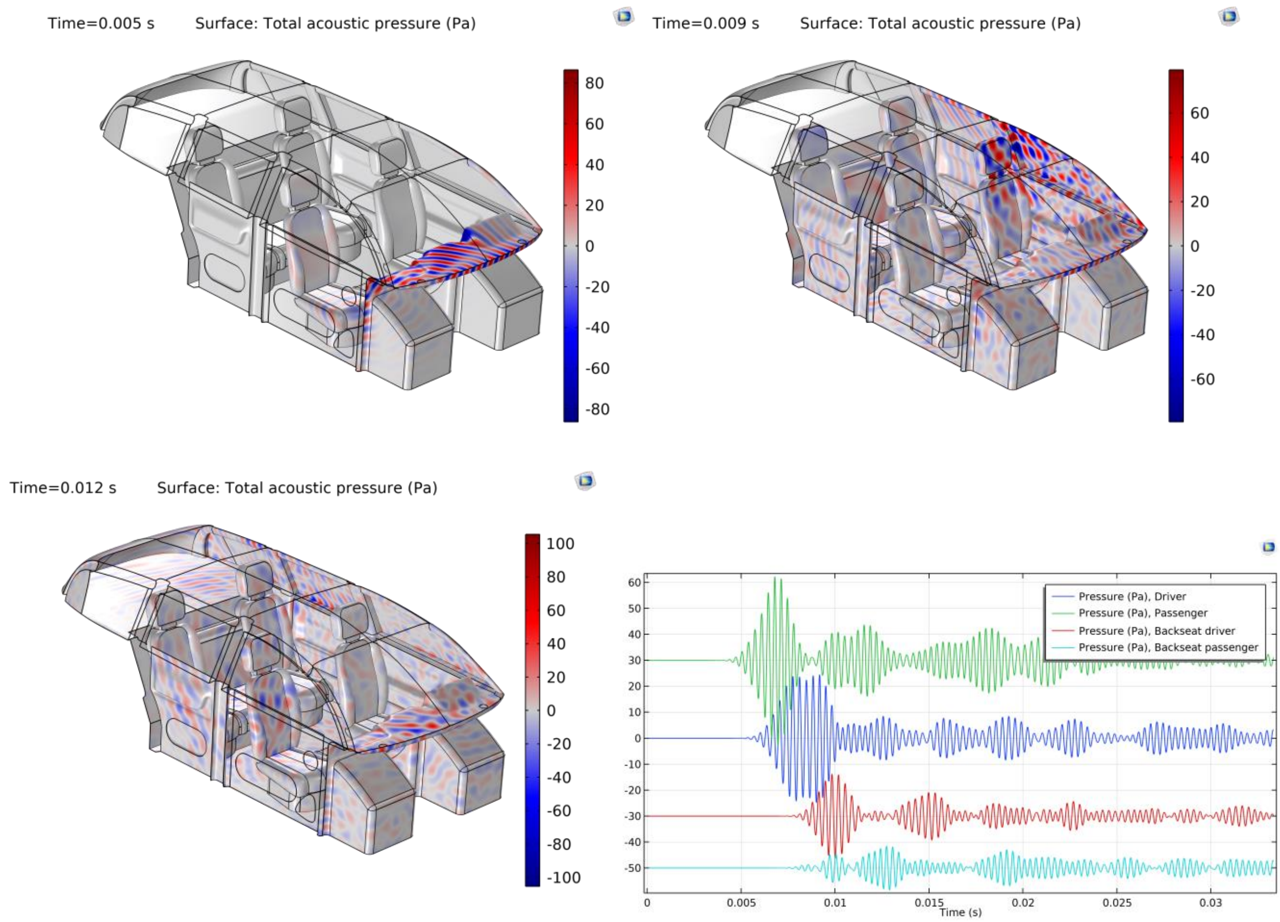

Figure 10: *Propagation of a pressure pulse inside the generic car cabin depicted at three different times along with the pressure over time in four listening positions.*

4 Conclusions

Various simulation strategies for modeling room acoustic applications are presented. The response of a room is first analyzed by a combined full wave and ray tracing method. The classical diffuse field absorption is replaced by an angle dependent impedance condition. Then the acoustics of a car cabin are analyzed using three approaches: a classical full wave strategy based on the finite element method using the latest solvers; a hybrid FEM-ray method where the classical point source approach is replaced by a more detailed sub-model and surface source; and using a full-wave time-domain model based on the dG-FEM time explicit method.

By combining methods and using sub-models, a more detailed acoustic characterization of rooms and listening spaces is possible. The applicability of classical approaches like ray tracing can be extended. But with the increase in modern computation power and development of new numerical methods, new trends are also emerging.

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